Level Attraction in Coupled Metamaterials

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What is a coupled system?

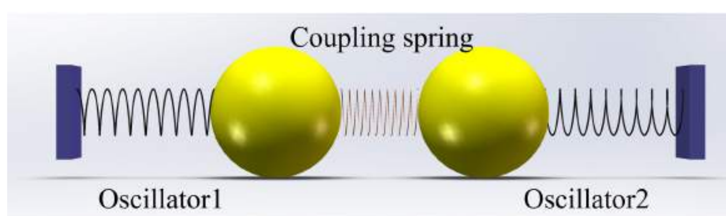

Figure 1. Coupled oscillators by a spring

- Coupled spring oscillators are one example of coupled systems. The energy can be transferred between each other. Consider the system is harmonic. The equation of motion is given by Newton's Second Law:

$$\ddot{x}_1 + \alpha\dot{x}_1 + \omega_1^2 x_1 + \kappa x_2 = Fe^{i\omega t}$$
$$\ddot{x}_2 + \beta\dot{x}_2 + \omega_2^2 x_2 + \kappa x_1 = 0 \qquad (1)$$

This equation of motion can be rewritten in a matrix by assuming the solution as a periodic form. Therefore, the behavior of the system can be acquired by solving the eigenvalue problem

$$\begin{pmatrix} \omega_1^2 - \omega^2 + i\omega\alpha & \kappa \\ \kappa & \omega_2^2 - \omega^2 + i\omega\beta \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix} \qquad (2)$$

Where ω_1, ω_2 are the intrinsic frequency of each oscillator, x_1, x_2 represent the relative positions away from equilibrium points, and κ is the coupling term that depends on the spring constant of the system.
The dispersion is given by:

$$\omega_{\pm} = \frac{1}{2}\left[\omega_1 + \omega_2 \pm \sqrt{(\omega_1 - \omega_2)^2 + 4\kappa^2}\right] \qquad (3)$$

This matrix model has been found a huge number of applications such as quantum information procession, and very useful explain the strong coupling between light and matter in various systems.

What is Level Attraction?

- The behavior of the coupled system can be solved analytically using the matrix model, for the system in Fig.1. The coupling term is real because of the spring. In this case, the eigenvalues of the system can be solved in the frequency domain and plotted in Fig.3 (left) which shows a repulsion behavior.

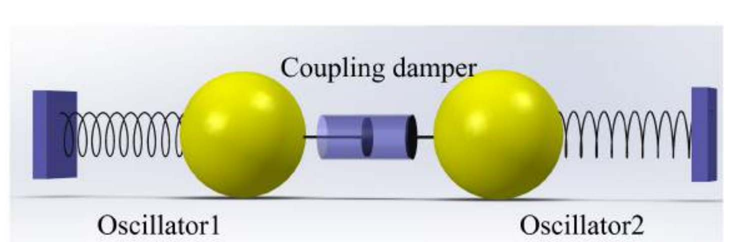

Figure 2. Coupled oscillators by a damper (dashpot)

- On the other hand, the coupling device is not limited to springs. Coupling depends on the relative velocity can be achieved by using a damper as the coupling device. The coupling term, in this case, would be imaginary and correspondingly produce an attraction behavior shown in Fig.3 (right).

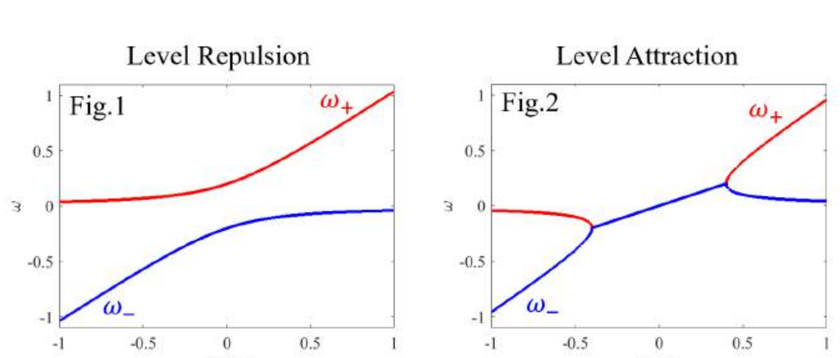

Figure 3. The dispersion relations of different coupled systems.

- Level attraction was firstly discovered in an optomechanical system in 2018[1]. Soon after, our group have demonstrated the dissipative coupling in a cavity-magnon system due to cavity Lenz effect [2]. The recent emergence of level attraction in diverse systems reveals its inherent nature in coupled systems.

Metamaterials:

- Metamaterials have certain engineered structures that are carefully designed by researchers, and thus physical properties are difficult to achieve in conventional, naturally occurring materials. Interesting properties are presented in metamaterials like negative refraction, cloaking device, absorber, antenna, etc.
- Level repulsion has been demonstrated in metamaterials [3]. This work shows the relative orientation of the metamaterial atoms are significantly effecting the strength of the coupling. Furthermore, the level attraction has been observed in planar structures [4]. Now comes the question, would level attraction also present in metamaterials?
- Inspired by the previous works, a cross cavity and split ring resonators were designed. Fig.4 (a-c) characterize the cross cavity. The signal was input from port 1 and collected from port 2. The end of the two arms of the cross cavity was short to the ground. Fig.4 (d-f) gives the properties of the split ring resonators (SRRs). For each of the SRR, the resonant frequency is fixed due to its geometry. Therefore, a series of SRRs with various radius has been fabricated.

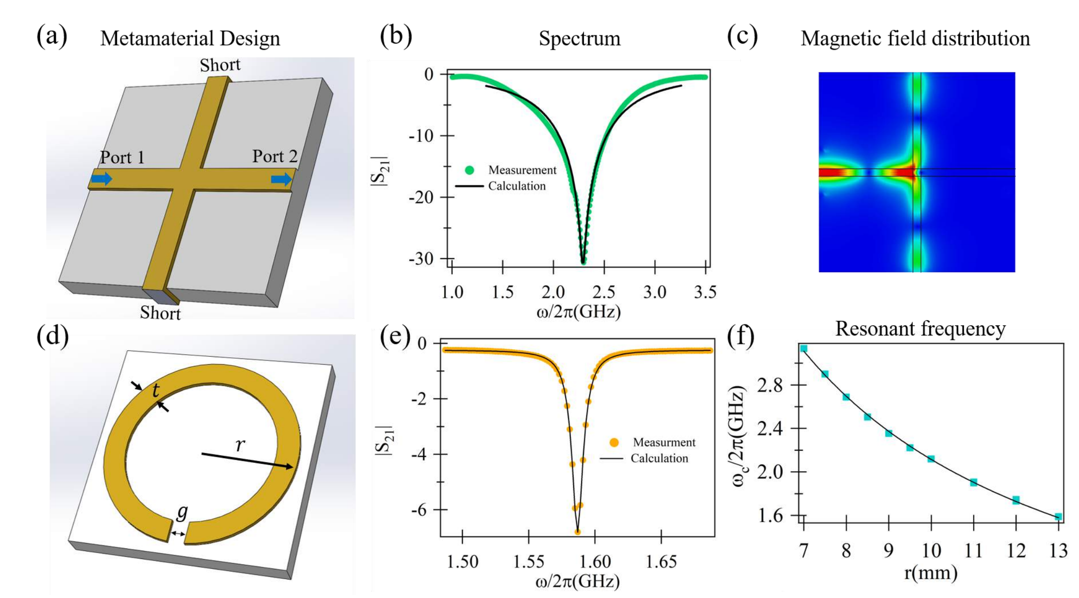

Figure 4. (a),(d) Structure design. (b),(e) spectra of the resonance with measurement and calculation. (c) the magnetic field distribution of cross cavity. (f) the resonance frequency for various radius SRR.

Level attraction in passive metamaterials:

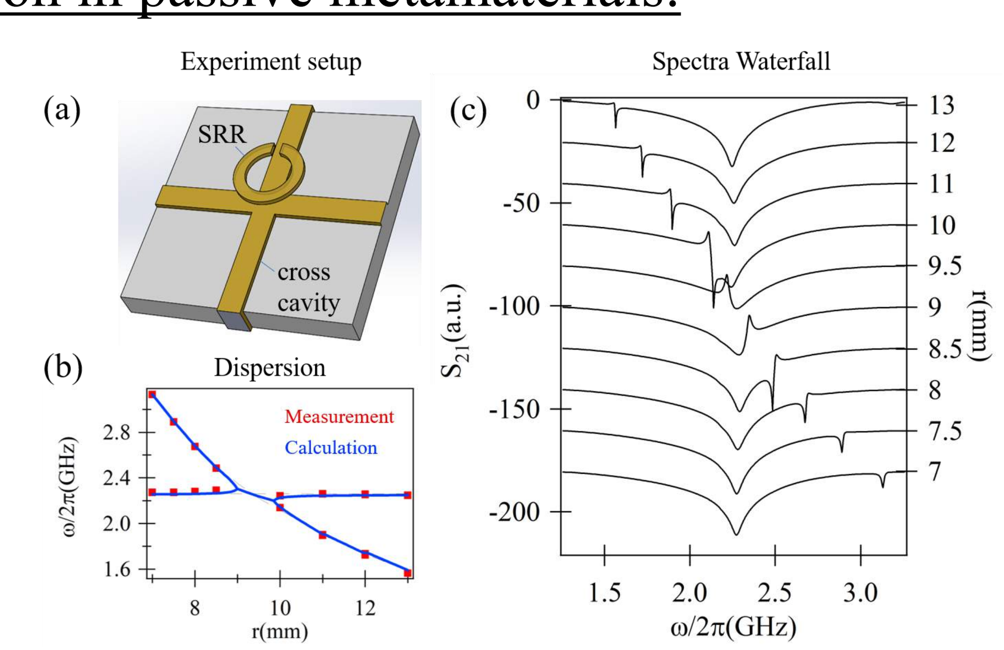

Figure 5. (a) Experiment setup. (b) The dispersion from measurement and calculation. (c) Waterfall plot as a function of radius SRRs.

- The experiment setup was given in Fig.5 (a) with the SRR placed above the cross cavity. By changing the SRRs, the waterfall plot of spectra can be obtained shown in Fig.5 (c). The relative position of the SRRs has been carefully controlled so that the node of SRRs is overlapping with the center of the cross to get the maximum coupling.
- The dispersion relation is given by the Fig.5 (b) with the points read out from experiment and the curve is calculated using the theoretical model we introduced in Eqn.(3). This shows the level attraction in passive metamaterials.

Level attraction using an active metamaterial:

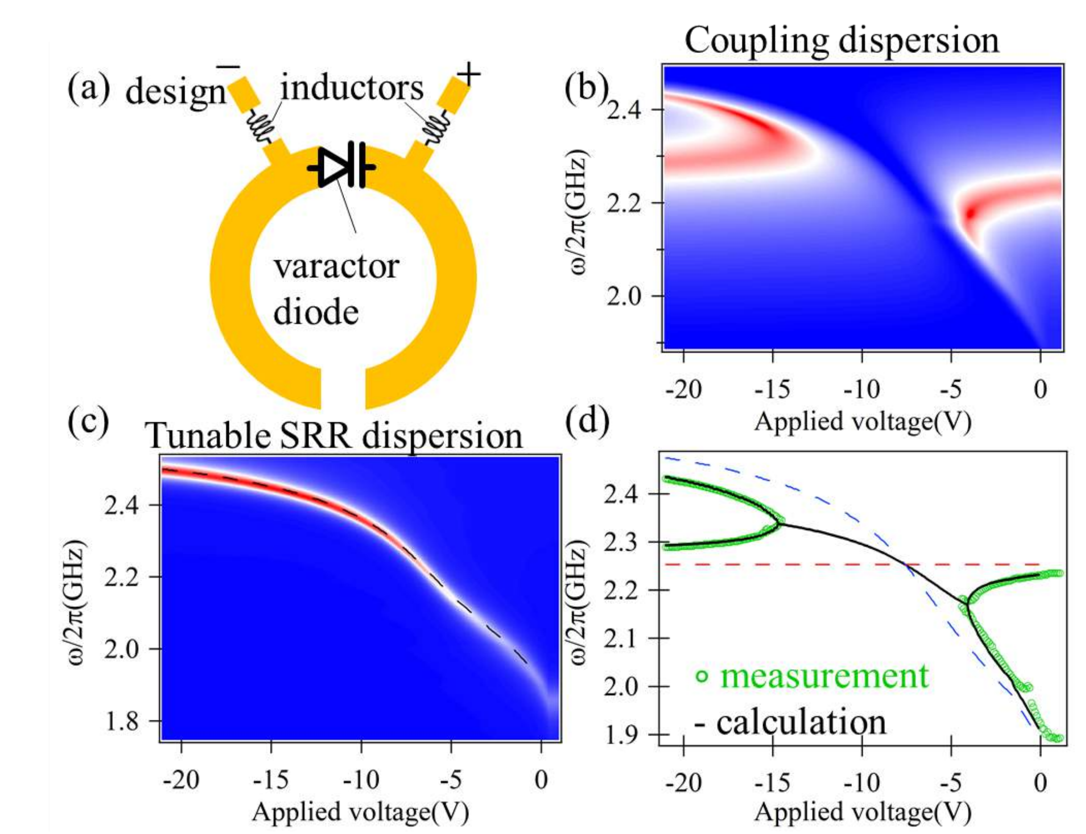

Figure 6.(a) Tunable SRR design. (b) The dispersion mapping of coupled system. (c) The dispersion of a single SRR as a function of the external voltage. (d) The measured and calculated results.

- The capacitance of the varactor can be changed by applied external voltage. By using this property , an active adjustable SSRs is designed and fabricated by integrating a varactor diode on the SRRs. The design was shown in Fig.6 (a). The tunable dispersion is demonstrated in Fig.6 (c).
- Coupling dispersion was acquired by placing the active metamaterial at the same position in Fig.5 (a) , the level attraction can be clearly observed in the mapping when the applied voltage is tuned. Moreover, the measured data and calculated curve are in good agreement. Therefore, the level attraction has been realized in metamaterials.

Significance

- This work expands the level attraction into field of metamaterials, and it offers more opportunity for advanced design of microwave devices.
- This system can be easily fabricated to on-chip devices which has the potential application on modules and may be more easily integrated into a lumped element system.

Acknowledgement

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Reference:

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