Time domain analysis of coupled system using numerical method

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Runge-Kutta methods： Numerical solution of Ordinary Differential equations (ODE)

All ODE can be rearranged in the form:

$$\dot{x}(t) = f(x(t), x)$$

With initial condition: $x(t = 0) = x_0$

We take a stepsize $h = t_{n+1} - t_n$, where t_n is the current system state and t_{n+1} is the system of after one time interval.

$$x(t_{n+1}) = x(t_n) + \int_{t_n}^{t_{n+1}} f\left(x(t), t\right) dt$$

Exact solution

$$x(t_{n+1}) = x(t_n) + \sum_{j=1}^{n} w_j k_j.$$

Numerical solution

based on the slope of the jth order of the interval

$$k_j = hf\left(x(t_n) + \sum_{i=1}^{j-1} \beta_{ji} k_i\right)$$

j – order of approximation
w_j – weight parameter
k_j – increasement

Runge-Kutta methods: Properties and example

We have weight parameter $\sum_{j=1}^{n} w_j = 1$

local truncation error - the error caused by one iteration step $\mathcal{O}(h^j)$

global truncation error - the cumulative error caused by many iteration step $\mathcal{O}(h^{j+1})$

Equation of motion for coherent coupling --- Mechanical system: spring coupled pendulums

$$\dot{x}(1) = x(2);$$

$$\dot{x}(2) = -2\lambda x(1) - (\omega_1^2 + 2\omega_1 J_1)x(2) + 2\omega_1 J_1 x(4);$$

$$\dot{x}(3) = x(4);$$

$$\dot{x}(4) = -2\lambda x(3) - (\omega_2^2 + 2\omega_2 J_2)x(2) + 2\omega_2 J_2 x(2),$$

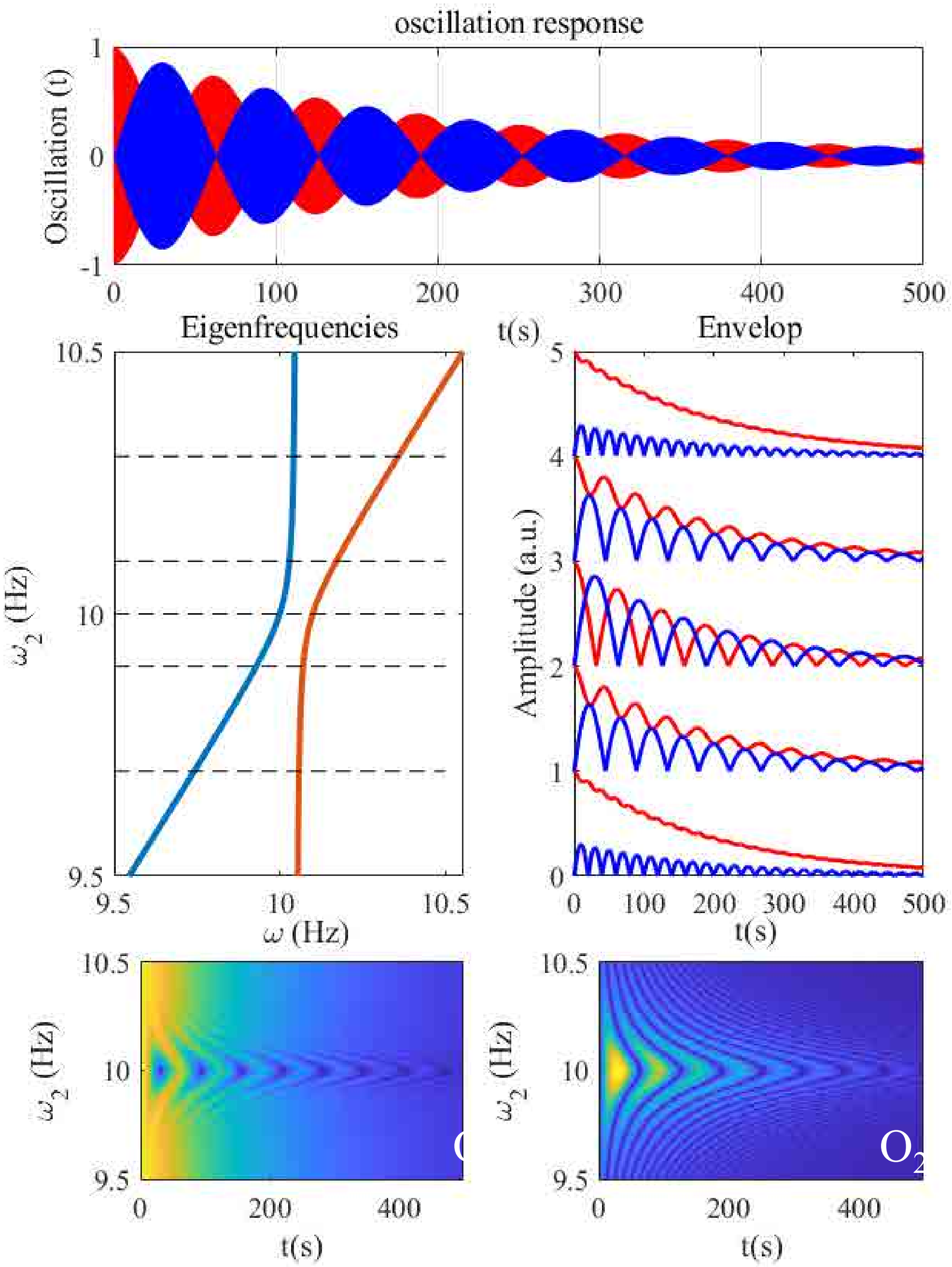
$\omega_1 = 10\ Hz$

$\omega_2 = 9\ - 11\ Hz$

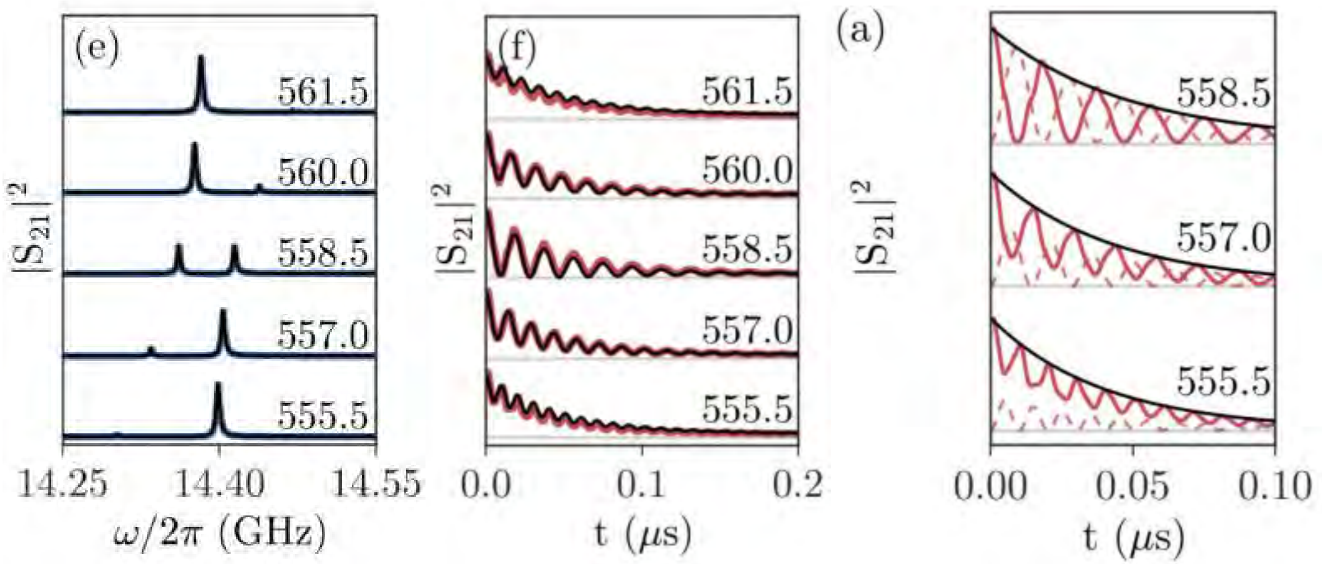
$\Delta\omega_1 = 0.01\ Hz$

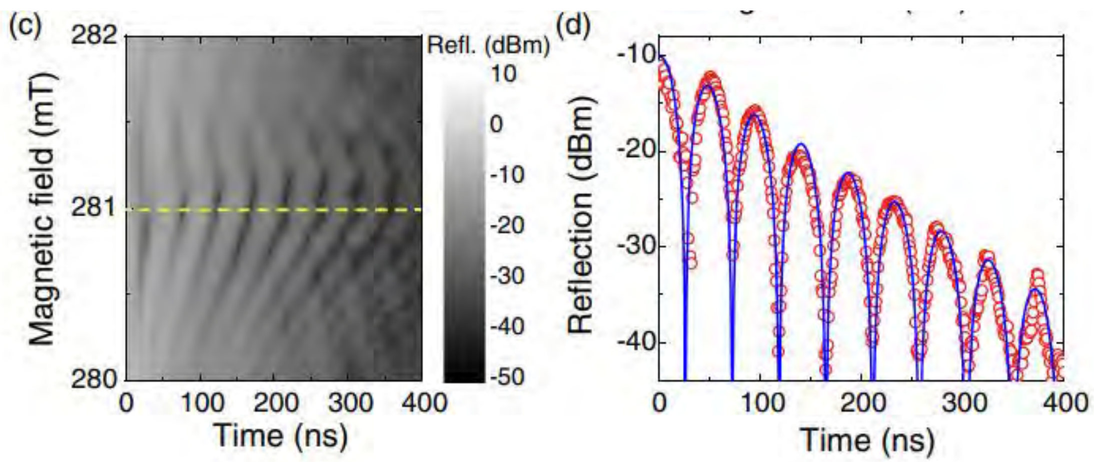
$\Delta\omega_2 = 0.01\ Hz$

$J = 0.05\ Hz$

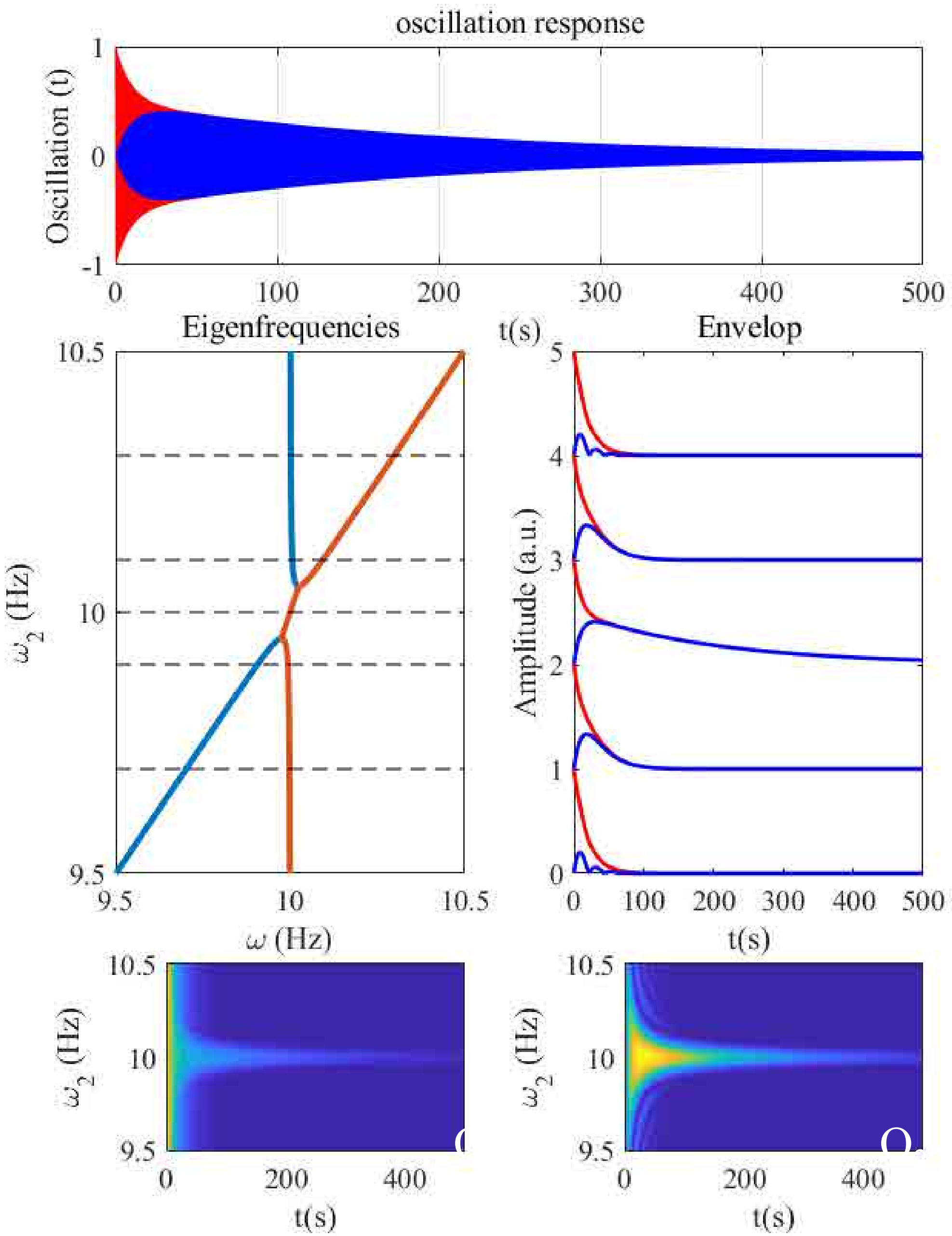


Match, Christophe, et al. "Transient response of the cavity magnon-polariton." *Physical Review B* 99.13 (2019): 134445.





Zhang, Xufeng, et al. "Strongly coupled magnons and cavity microwave photons." *Physical review letters* 113.15 (2014): 156401.



Equation of motion for dissipative coupling

$$\dot{x}(1) = x(2);$$

$$\dot{x}(2) = (-2\lambda_1 - 2\Gamma)x(1) - \omega_1^2 x(2) + 2\Gamma_1 x(3);$$

$$\dot{x}(3) = x(4);$$

$$\dot{x}(4) = (-2\lambda_2 - 2\Gamma)x(3) - \omega_2^2 x(2) + 2\Gamma_2 x(1),$$

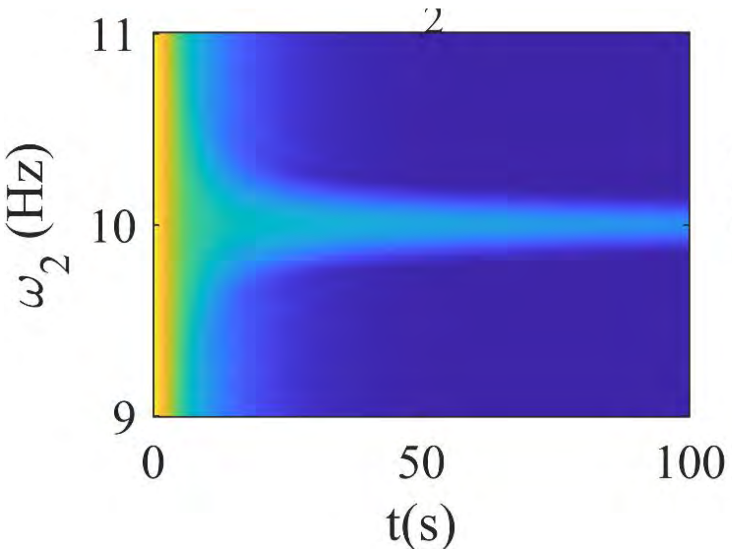
$\omega_1 = 10\ Hz$

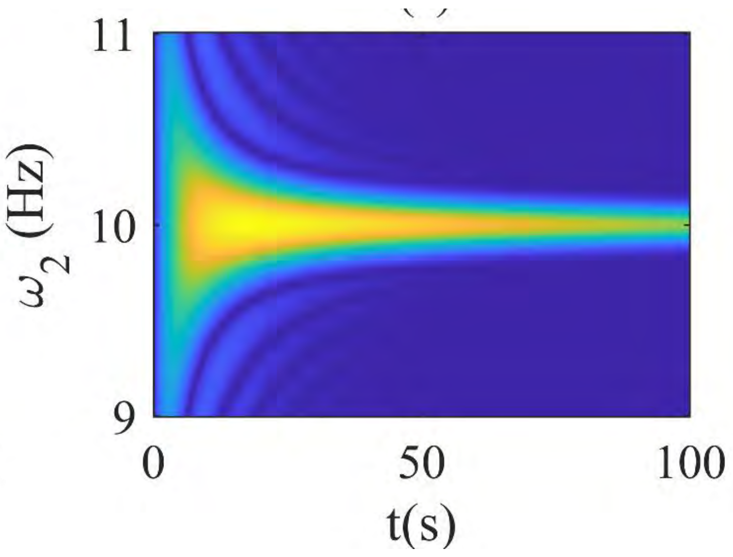
$\omega_2 = 9\ - 11\ Hz$

$\Delta\omega_1 = 0.1\ Hz$

$\Delta\omega_2 = 0.1\ Hz$

$\Gamma = 0.1\ Hz$

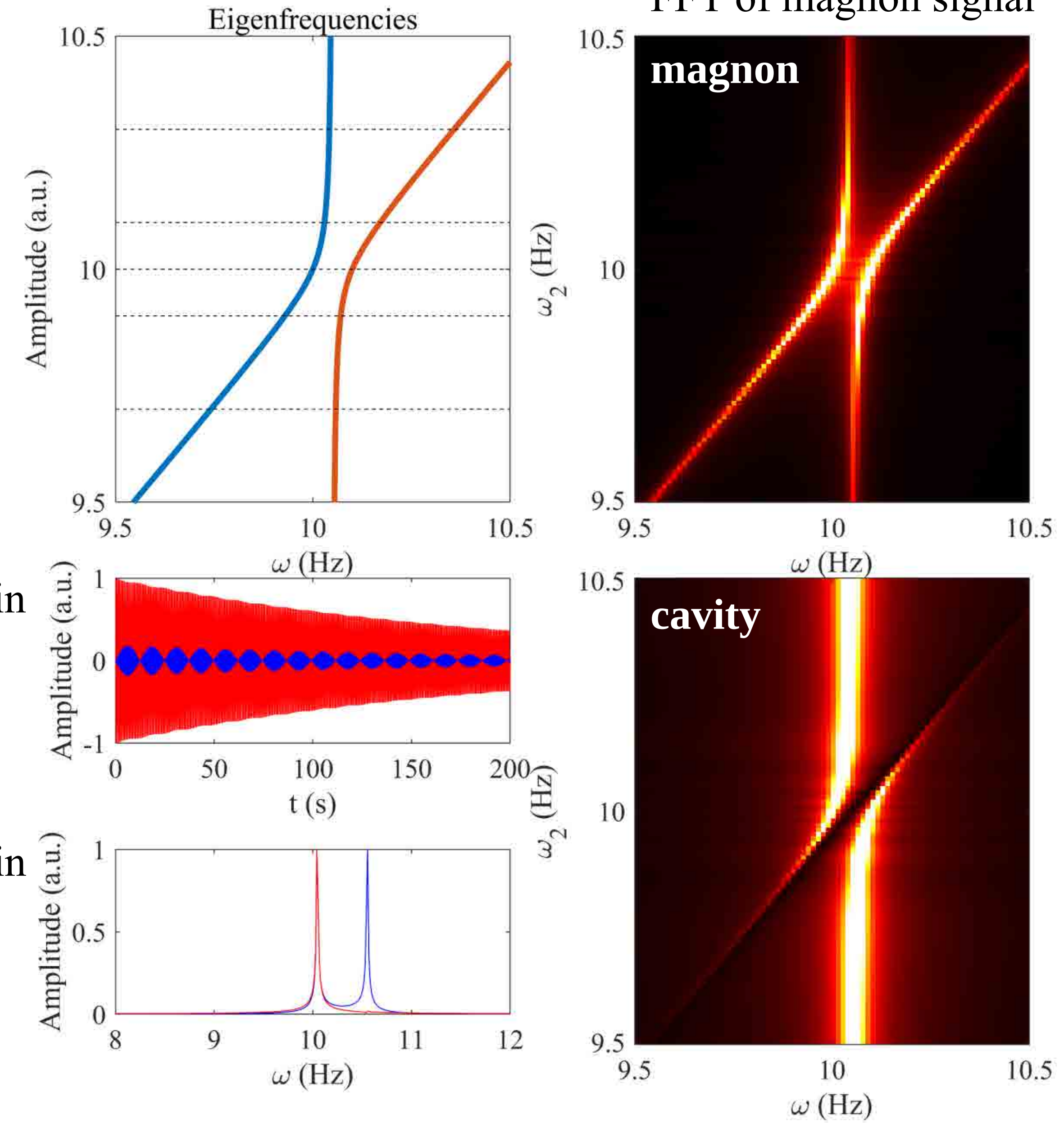




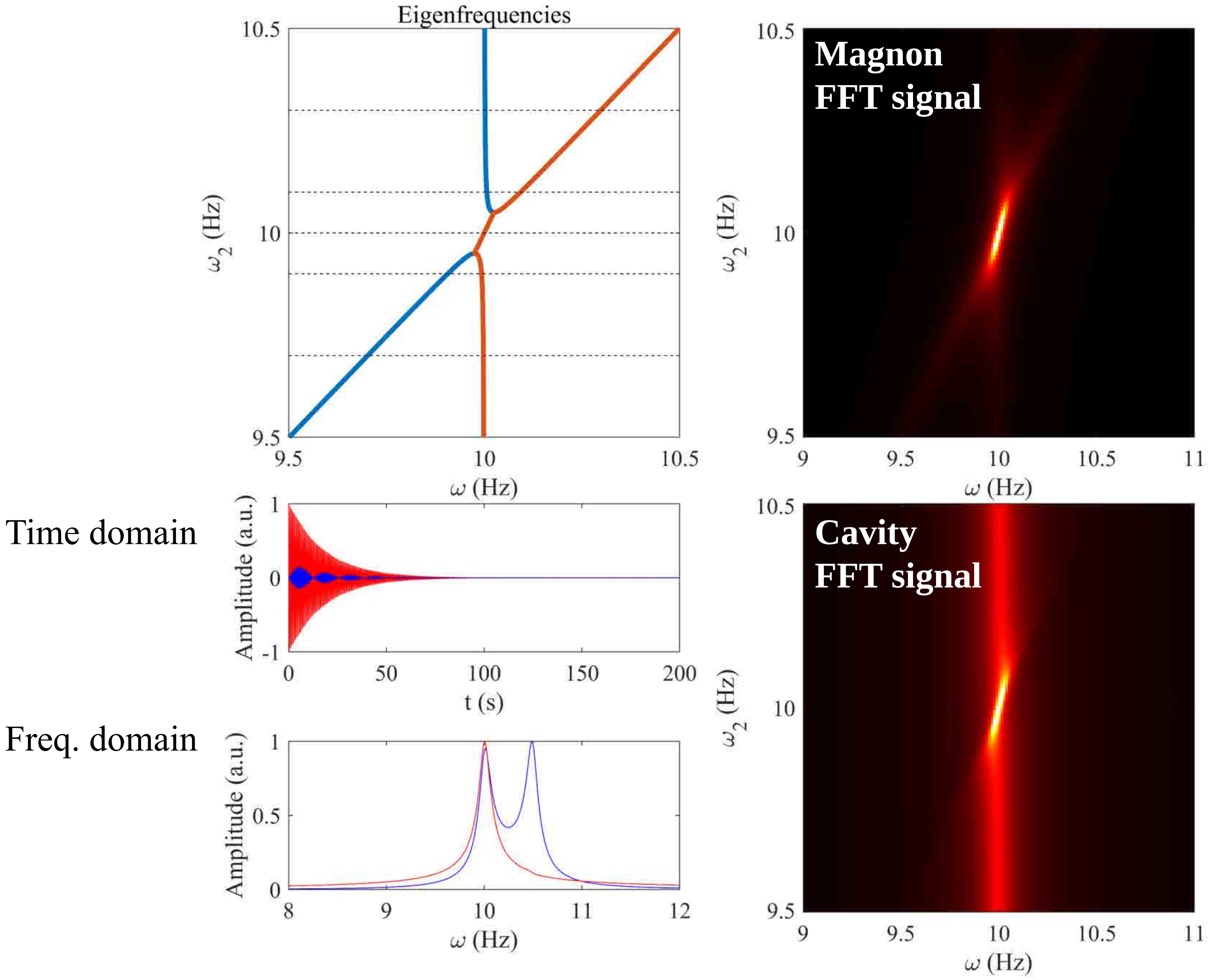
FFT of magnon signal

Time domain

Freq. domain



FFT of cavity signal


Eigenfrequencies
Magnon
FFT signal
Cavity
FFT signal
Time domain
Freq. domain
ω_2 (Hz)
ω (Hz)
Amplitude (a.u.)
t (s)

Summary

- Better understand on coherent coupling, especially on the magnon perspective.
- Provide preliminary knowledge on dissipative coupling, and provide time domain signal for both cavity and magnon.

Future work:

- Build the model for dissipative coupling for electromagnetic systems.